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Composite Sliding Mode Control for a Free-Floating Space Rigid-Flexible Coupling Manipulator System

Regular Paper

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Abstract The flexible space manipulator is a highly nonlinear and coupled dynamic system. This paper proposes a novel composite sliding mode control to deal with the vibration suppression and trajectory tracking of a free-floating space rigid-flexible coupling manipulator with a rigid payload. First, the dynamic equations of this system are established by using Lagrange and assumed mode methods and in the meantime this dynamic modelling allows consideration of the modelling errors, the external disturbance and the vibration damping of a flexible link. Then, in modal space, the problems of the manipulator system's trajectory tracking and the vibration suppression are discussed by using the composite control approach, which combines a non-singular terminal sliding mode control (NTSMC) with an active vibration suppression control (AVSC). The NTSMC uses a fuzzy logic output instead of the symbol item, which smoothes the control signal, thereby inhibiting the chattering of the sliding mode control. Compared with common sliding mode control (SMC), the approach not only can reduce the chattering of the sliding mode control, but also can eliminate the singular phenomenon of the system's control input. In addition, it can assure the trajectory tracking and the vibration suppression. Many space missions can benefit from this modelling system, such as autonomous docking of satellites, rescuing and satellite servicing. Finally, the numerical simulations were carried out, which confirmed the effectiveness of these methods.

Keywords Space Rigid-flexible Coupling Manipulator, Non-Singular Terminal Sliding Mode Control, Active Vibration Control, Trajectory Tracking

1. Introduction

With the rapid development of space technology, there has been great interest in the design and control of space manipulators with flexible links and bases. These have a number of advantages: light weight, small driving force and low energy consumption etc. However, the structural flexibility inevitably causes elastic deflection and vibration. So the control problem for flexible manipulators is much more complex than the equivalent one for rigid manipulators. A flexible manipulator

controller not only must achieve the same motion objectives as a rigid manipulator, but also must stabilize the vibration that is naturally excited. Furthermore, a primary problem concerns the motion coordination between the spacecraft and the manipulator. In fact, due to the lack of a stationary base, the free-floating spacecraft will react to the motions of the attached manipulator. If the inertia of the manipulator, with respect to that of the spacecraft, is non-negligible, the application of conventional control techniques for a ground-fixed manipulator becomes unfeasible and different strategies have to be pursued. A flexible space manipulator is a highly nonlinear and coupled dynamic system. Many techniques in dynamic modelling of space robots have been developed [1, 2, 3]. The assumed mode method to describe the elastic deformation has been used in [4] and the dynamic model of a flexible dual-arm space robot is built by the Lagrange approach. The problem of the dynamics and control of a flexible space robot capturing a static target was presented in [5]. The dynamic model of the robot system is derived with Lagrangian formulation. In [6] the problem of the generalized contact forces between the space robot end-effecter and the target satellite as internal holonomic and nonholonomic constrains has been considered. Nowadays, many researchers have concentrated on developing control strategies for design and implementation in various applications of flexible manipulators. In several pieces of work [7, 8, 9] neural network based control schemes have been proposed to obtain controllers with adaptation capabilities for the uncertainties for space manipulators. Classical adaptive control schemes have been proposed [10, 11] and other control schemes, like robust control [12, 13, 14] and namely intelligent control [15], can be found.
Variable structure control with a sliding mode is a powerful robust nonlinear control technique that has been intensively developed during the last 35 years [16, 17]. The term "variable structure system" (VSS) first made its appearance in the late 1950s. Since that time, the first expectations of such systems have naturally been re-evaluated and their real potential has been revealed. Novel research directions have originated due to the appearance of novel classes of control problems, novel mathematical methods and novel control principles. The sliding mode design approach usually consists of two steps [18, 19]. First, the switching surface is designed such that the system motion in sliding mode satisfies design specifications. Second, a control function is designed making the switching surface attractive to the system state. In some work [20, 21], the non-singular terminal sliding mode controllers have been proposed for flexible and rigid manipulators, respectively. A novel design approach of a multiple input multiple output adaptive fuzzy terminal sliding-mode controller for robotic manipulators is described in [22], which does not require detailed system parameters for the presented controller. In [23] a robust control approach is developed to control a robot in the task space using a sliding mode by support of feedback linearization control and a back-stepping method. [24] proposed a PID integral variable structure regulation controller for robot manipulators.

In this paper a novel composite sliding mode control approach for a free-floating space rigid-flexible coupling manipulator with a rigid payload are proposed. The dynamic model of the manipulator is established by considering the modelling errors, the external disturbance and the vibration damping of a flexible link, based on Lagrange and assumed mode methods, as well as conservation of momentum theorem. The composite control approach combines NTSMC with AVSC for trajectory tracking and vibration suppression. The general sliding mode control inevitably causes chattering and may cause a singular phenomenon. Compared with the SMC, the novel NTSMC with the fuzzy logic inference can inhibit the chattering of the sliding mode control, at the same time the singular phenomenon can be eliminated for the system's control input. Finally, numerical simulations are performed to demonstrate the effectiveness of the proposed methods.

2. Dynamic model description for the manipulator system

In this section we introduce the dynamic model of a free-floating space rigid-flexible coupling manipulator with three links. It includes a base, three links and a rigid payload. Among them, L_1 and L_2 are rigid links and L_3 is a flexible link. Let $\boldsymbol{X_c} = [x_c \ y_c \ \theta_c]^T$ be the pose of the payload, $\boldsymbol{X_b} = \left[x_b \quad y_b \quad \theta_b\right]^T$ be the pose of the base, θ_c is the attitude angle of the base and θ_b is the attitude angle of the base. The coordinate systems in Fig.1 are defined as follows:

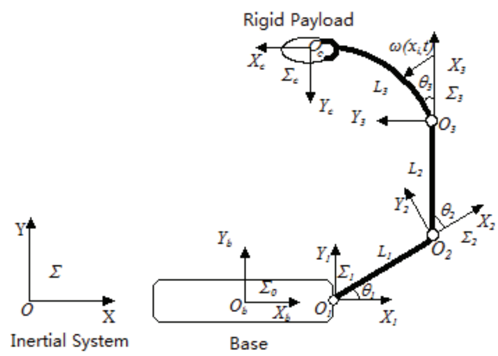

Figure 1. An articulated three-link rigid-flexible coupling space manipulator; Σ-OXY—The inertial coordinate system. Σ_o-$O_bX_bY_b$—The base coordinate system. Σ_i-$O_iX_iY_i$—The fixed in the i[th] link coordinate system. Σ_c-$O_cX_cY_c$—The payload coordinate system fixed to the centre of mass of the payload.

The link coordinate system is established in accordance with Denavit-Hartenberg rules. The elastic deformation mode is described as $\omega(x_3,t)$ by the assumed mode method:

$$\omega(x_3,t)=\sum_{i=1}^{n}\sin(i\pi x_3 / L_3)\cdot\eta_i(t) \tag{1}$$

where n is the mode's order($n=2$), $\eta_i(t)$ is the flexible link's generalized deformation variable and x_3 is the X_3 direction coordinate in the coordinate system of the flexible link.

The total momentum of the system is given by:

$$T=T_b+\sum_{i=1}^{3}T_i+T_c \tag{2}$$

where $T_b=\frac{1}{2}\mathrm{I}_b\dot{\theta}_b^{\,2}+\frac{1}{2}m_b\dot{x}_b^{\,2}+\frac{1}{2}m_b\dot{y}_b^{\,2}$ is the kinetic energy of the base, $T_i=\frac{1}{2}\mathrm{I}_i\dot{\theta}_i^2+\frac{1}{2}\int_0^{L_i}\rho_i\dot{R}_i^T\dot{R}_i dx_i$ is the kinetic energy of the i[th] link, $T_c=\frac{1}{2}\mathrm{I}_c\dot{\theta}_c^{\,2}+\frac{1}{2}m_c\dot{x}_c^{\,2}+\frac{1}{2}m_c\dot{y}_c^{\,2}$ is the kinetic energy of the payload, where ρ_i is the linear density of the i[th] link and x_i is the distance from any point of the i[th] link to the i[th] link coordinate's origin. $\dot{R}_i$ represents the speed of any point on the i[th] link. m_b is the mass of the base. m_c is the mass of the payload. I_b is the inertia moment of the base. I_c is the inertia moment of the payload.

Moreover, the total potential energy of the system is given by:

$$U=U_1+U_2+U_3 \tag{3}$$

where $U_i(i=1,2)$ is the potential energy of the rigid links $L_i(i=1,2)$. As the system is in micro-gravity space, the gravitational potential energy can be ignored. The potential energy of the system is only generated by the elastic deformation of the flexible-link. Then $U_1=0, U_2=0$.

The flexible link's potential energy can be written as:

$$U_3=\frac{1}{2}\int_0^{L_3}EI(\frac{\partial^2\omega(x_3,t)}{\partial x_3^{\,2}})^2dx_3 \tag{4}$$

where EI is the module of elasticity.

According to Eq. (2) and Eq. (3), the Lagrange function is given by:

$$L=T-U \tag{5}$$

Based on the principle of D'Alembert-Lagrange, the Lagrange equations are given by:

$$\begin{cases}\frac{d}{dt}[\frac{\delta L}{\delta\dot{\theta}_i}]-\frac{\delta L}{\delta\theta_i}=\tau_i \quad (i=1,2)\\ \frac{d}{dt}[\frac{\delta L}{\delta\dot{\theta}_i}]-\frac{\delta L}{\delta\theta_i}=\tau_i-J^{\mathrm{T}}F \quad (i=3)\\ \frac{d}{dt}[\frac{\delta L}{\delta\dot{\eta}_i}]-\frac{\delta L}{\delta\eta_i}=0 \quad (i=1,2)\end{cases} \tag{6}$$

where τ_i is the joint torque, $J\in R^{3\times3}$ is the manipulator's Jacobian matrix and $F\in R^{3\times1}$ is the force vector exerted by the end-effector on the payload at the grasp point.

According to Eq. (6), the dynamic equations of the system are given by:

$$M_{11}\ddot{X}_b+M_{12}\ddot{\theta}+M_{13}\ddot{Q}+D_{11}\dot{X}_b+D_{12}\dot{\theta}+D_{13}\dot{Q}=\tau-J^TF \tag{7}$$

$$M_{21}\ddot{X}_b+M_{22}\ddot{\theta}+M_{23}\ddot{Q}+D_{21}\dot{X}_b+D_{22}\dot{\theta}+KQ=0 \tag{8}$$

where $M_{ij}(i=1,2, j=1,2,3)$ are the generalized mass matrices, D_{ij} $(i=1,2, j=1,2,3)$ are the damp matrices, K is the stiffness matrix of the flexible links and $Q=\begin{bmatrix}\eta_1 & \eta_2\end{bmatrix}^T=\begin{bmatrix}Q_1 & Q_2\end{bmatrix}^T$ is the generalized deformation variable. $\theta=\begin{bmatrix}\theta_1 & \theta_2 & \theta_3\end{bmatrix}^T$ is the joint angle vector of the links and $\tau=\begin{bmatrix}\tau_1 & \tau_2 & \tau_3\end{bmatrix}^T$ is the joint torque vector.

According to the Newton-Euler formula, dynamic equations of the payload are given by:

$$m_c\ddot{r}_c=f_o \tag{9}$$

$$I_c\ddot{\theta}_c+\dot{\theta}_c\times(I_c\dot{\theta}_c)=N_o \tag{10}$$

where m_c is the mass of the payload, I_c is the inertia moment of the payload, f_o is the resultant force between the end-effector and the payload and N_o is the resultant moment between the end-effector and payload. r_c is the position vector of the payload centre with respect to the inertial coordinate system.

According to Eq. (9) and Eq. (10) , the dynamic equation of the payload can be written as:

$$D(X_c)\ddot{X}_c+C(X_c,\dot{X}_c)=GF=F_o \tag{11}$$

where $D(X_c)=\begin{bmatrix}m_cE_2 & 0\\ 0 & I_c\end{bmatrix}\in R^{3\times3}$, $E_2\in R^{2\times2}$ is the unit matrix and $C(X_c,\dot{X}_c)=[0\;0\;(\dot{\theta}_c\times I_c\dot{\theta}_c)^T]^T$, $G=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -Py & Px & 1\end{bmatrix}\in R^{3\times3}$ is the grasping matrix. Px , Py are the components of the position vectors with respect to the payload coordinate system. $F_o=[f_o^T,N_o^T]^T\in R^{3\times1}$. The end-effector force $F=G^{+}F_o$ that satisfies Eq. (12) can be decomposed into two orthogonal components [25], one contributes to the motion of the payload and the other produces the grasping force $F_g \in R^{3\times1}$ between the end-effector and the payload. $G^{+} = G^T(GG^T)^{-1} \in R^{3\times3}$ is the pseudo-inverse matrix of G.

The end-effector force F can be expressed as:

$$F = G^{+}(D(X_c)\ddot{X}_c + C(X_c, \dot{X}_c)) + F_g \tag{12}$$

The relationship between $\dot{X}_c$, $\dot{X}_b$ and $\dot{\theta}$ is given by:

$$\dot{X}_c = J_{cb}\dot{X}_b + J_{cq}\dot{\theta} \tag{13}$$

where J_{cb} is the Jacobian matrix for the payload with respect to the base and J_{cq} is the Jacobian matrix for the payload with respect to the joint.

We assume the space manipulator system is operated in free-floating mode, so that linear and angular momenta are conserved. On the assumption that total linear and angular momenta are zero, so we can obtain:

$$H_1\ddot{X}_c + H_2\ddot{Q} + H_3\dot{X}_c + H_4\dot{Q} + H_5Q = \tau - J^T F \tag{14}$$

$$\ddot{Q} = -N_2^{-1}(N_1\ddot{X}_c + N_3\dot{X}_c + N_4\dot{Q} + N_5Q) \tag{15}$$

$$A_c\ddot{X}_c + B_c\dot{X}_c + C_c\dot{Q} + D_cQ + E_c = \tau - J^T F_g \tag{16}$$

where $H_i\ (i = 1\cdots5)$, $N_i\ (i = 1\cdots5)$, A_c, B_c, C_c, D_c, E_c are the coefficient matrices, respectively. Eq. (16) is the dynamic equation of the system.

In the actual modelling process there are measurement errors of model parameters and the joint friction, as well as other factors of uncertainty, which lead to a systematic modelling error. The system in practice will be subjected to a variety of external disturbances, so Eq. (14) and Eq. (15) can be rewritten as:

$$H_1\ddot{X}_c + H_2\ddot{Q} + H_3\dot{X}_c + H_4\dot{Q} + H_5Q + d_1 + d_2 = \tau - J^T F \tag{17}$$

$$\ddot{Q} = -N_2^{-1}(N_1\ddot{X}_c + N_3\dot{X}_c + N_4\dot{Q} + N_5Q + d_3) \tag{18}$$

where d_1 is the model error for the system, d_2 is the external disturbance and d_3 is the vibration damping of the flexible link.

According to Eq. (17) and Eq. (18), the dynamic equation (16) can be modified to:

$$A_k\ddot{X}_c + B_k\dot{X}_c + C_k\dot{Q} + D_kQ + E_k = \tau - J^T F_g \tag{19}$$

where $A_k = H_1 - H_2N_2^{-1}N_1 + J^TG^{+}D(X_c)$,
$B_k = H_3 - H_2N_2^{-1}N_3$, $C_k = H_4 - H_2N_2^{-1}N_4$,
$D_k = -H_2N_2^{-1}N_5$, $E_k = J^TG^{+}C(X_c, \dot{X}_c) + d_1 + d_2 - H_2N_2^{-1}d_3$

3. Sliding mode control for the manipulator system

In this section the controller for robotic manipulators combines the advantages of the non-singular terminal sliding mode control and the fuzzy inference mechanism.

Eq. (19) can be written as the following state equation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1 + u_1 \end{cases} \tag{20}$$

where $x_1 = X_c$, $x_2 = \dot{X}_c$,
$f_1 = -A_k^{-1}(B_k\dot{X}_c + C_k\dot{Q} + D_kQ + E_k)$, $u_1 = A_k^{-1}(\tau - J^T F_g)$.

It is assumed that $f_1 = \hat{f}_1 + \Delta f$, where $\hat{f}_1$ is an estimated value of f_1, Δf is the model uncertainty with modelling error and the external disturbance and $|\Delta f| \le \alpha$, α is a positive constant defined as the upper boundary of Δf.
Let's define the tracking error:

$$e = x_1 - x_{1d} = X_c - X_c^d \tag{21}$$

where $x_{1d} = X_c^d$ is the desired payload's position.

3.1 SMC law

To obtain the finite time convergence of the system tracking error, the sliding surface is defined as:

$$s = \dot{e} + K_1 e \tag{22}$$

where $K_1 = diag(\lambda_1, \lambda_2, \lambda_3)$, in which $\lambda_i (i = 1,2,3)$ is a positive constant.

The SMC(22) derivative along the state Eq. (20) of the system dynamics is:

$$\begin{aligned} \dot{s} &= \ddot{X}_c - \ddot{X}_c^d + K_1\dot{e} \\ &= f_1 + u_1 - \ddot{X}_c^d + K_1\dot{e} \end{aligned} \tag{23}$$

The controller can then be expressed as:

$$u_1 = -\hat{f}_1 + \ddot{X}_c^d - K_1\dot{e} - K_s \cdot \text{sgn}(s) \tag{24}$$

where $\text{sgn}(s)$ is the sign function of s, i.e., $\text{sgn}(s) = 1$ if s>0; $\text{sgn}(s) = 0$ if s=0; $\text{sgn}(s) = -1$ if s<0. The switching gain K_s is a positive constant and $K_s \ge \alpha$.

Substituting Eq. (22) into Eq. (23) yields:

$$\dot{s} = \Delta f - K_s \cdot \text{sgn}(s) \tag{25}$$

Obviously, $s\dot{s} \le 0$. Under the action of the controller u_1, the sliding mode is subsistent and accessible.

3.2 NTSMC law

To obtain the finite time convergence of the system tracking error, the sliding surface is defined as:

$$s = e + K_e^{-1}\dot{e}^{p/q} = e + F_1(\dot{e}) \quad (26)$$

where $K_e = diag(\lambda_1, \lambda_2, \lambda_3)$, in which $\lambda_i (i = 1,2,3)$ is a positive constant, p,q are odd numbers and $q < p < 2q$. $\dot{e}^{p/q} = [\dot{e}_1^{p/q}, \dot{e}_2^{p/q}, \dot{e}_3^{p/q}]^T$, $F_1(\dot{e}) = K_e^{-1}\dot{e}^{p/q}$.

The NTSMC(26) derivative along the state Eq. (20) of the system dynamics is:

$$\dot{s} = \dot{e} + \frac{p}{q} K_e^{-1} \dot{e}^{(p/q-1)} \ddot{e} \quad (27)$$

So, the controller can be expressed as:

$$u_1 = -\hat{f}_1 + \ddot{X}_c^d - \frac{p}{q} K_e \dot{e}^{2-q/p} - K_s \cdot \mathrm{sgn}(s)$$

$$= -\hat{f}_1 + \ddot{X}_c^d - F_2(\dot{e}) - K_s \cdot \mathrm{sgn}(s) \quad (28)$$

where K_s is a switching gain and $K_s \ge \alpha$, $F_2(\dot{e}) = \frac{p}{q} K_e \dot{e}^{2-q/p}$.

Remark: when $\dot{e} \ne 0, e = 0$, $u_1 \to \infty$, namely a singular phenomenon has occurred.

Theorem 1: For the input-output system, if the sliding surface of the non-singular terminal sliding mode is selected as Eq. (26) and the controller is designed as Eq. (28), then the system's state errors will converge to zero in a finite time.

Proof: The Lyapunov function is chosen as:

$$V = \frac{1}{2} s^T s \quad (29)$$

The derivative of the Lyapunov function, with respect to time, is obtained as:

$$\dot{V} = s^T \dot{s}$$

$$= (e + F_1(\dot{e}))^T \frac{F_1(\dot{e})}{\dot{e}} (\Delta f - K_s \mathrm{sgn}(e + F_1(\dot{e})))$$

$$\le (e + F_1(\dot{e}))^T \frac{F_1(\dot{e})}{\dot{e}} (\alpha - K_s \mathrm{sgn}(e + F_1(\dot{e})))$$

where K_s is a switching gain and $K_s \ge \alpha$.

Thus $\dot{V} \le 0$ for all t and the system is stable and the system's state errors will converge to zero in a finite time.

The NTSMC not only can guarantee the system states reach a terminal sliding surface in a finite time, but also can eliminate the singular phenomenon for the system.

3.3 Jitter suppression and force control of the manipulator system

When $s = 0$, switching will bring about jitter. For jitter suppression, a saturation function $sat(s/\delta)$ can be used, instead of the sign function $\mathrm{sgn}(s)$.

$$sat(x) = \begin{cases} x, & |x| < 1 \\ \mathrm{sgn}(x), & |x| \ge 1 \end{cases},$$ δ is the thickness of the boundary layer.

In this section, fuzzy inference rules are used to adjust the fuzzy quantization factor of the switching function. On the one hand it can guarantee the stability of the entire system, on the other hand it ensures that the system has good transient performance. We set:

$$K_s \cdot \Delta u = -K_s \cdot sat(s) \quad (30)$$

According to Eq. (30), $\begin{cases} \Delta u < 0, & s > 0 \\ \Delta u > 0, & s < 0 \end{cases}$. Through fuzzy set theory to determine Δu, let it substitute the switch function for $sat(s)$, in order to make the control signal smooth.

First, we select positive constant β and normalize s, assuming $\hat{s} = \beta s$, and define $\hat{s}$ as the input variable of the fuzzy system and Δu as the output variable of the fuzzy system.

Second, both $\hat{s}$ and Δu are partitioned into seven fuzzy subsets: negative big (NB), negative middle(NM), negative small(NS), zero (ZE), positive small(PS), positive middle(PM) and positive big (PB). The triangular shape membership function of $\hat{s}$ is shown in Fig. 2. The singleton membership function of Δu is shown in Fig. 3.

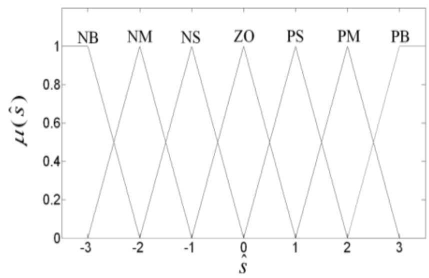

Figure 2. Input membership function

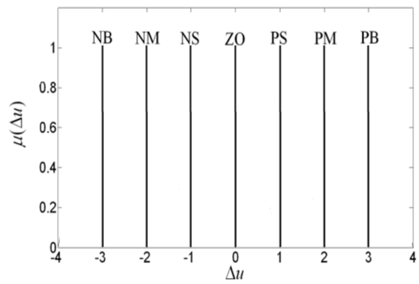

Figure 3. Output membership function

According to Fig. 2 and Fig. 3, the fuzzy rules can be determined as follows:

IF $\hat{s}$ is NB, THEN Δu is PB;
IF $\hat{s}$ is NM, THEN Δu is PM;
IF $\hat{s}$ is NS, THEN Δu is PS;
IF $\hat{s}$ is ZO, THEN Δu is ZO;
IF $\hat{s}$ is PS, THEN Δu is NS;
IF $\hat{s}$ is PM, THEN Δu is NM;
IF $\hat{s}$ is PB, THEN Δu is NB.

Choosing the weighted average defuzzification, the output of the fuzzy inference system can be written as:

$$K_s \cdot \Delta u = K_s \cdot \frac{\sum_{i=-3}^{3} \mu_i(\hat{s})(-i)}{\sum_{i=-3}^{3} \mu_i(\hat{s})} \qquad (31)$$

where $\mu_i(\hat{s})$ is the strength of the i[th] rule and $(-i)$ is the associated single membership function of Δu.

According to $u_1 = A_k^{-1}(\tau - J^T F_g)$, we define the controller as τ:

$$\tau = A_k u_1 + \tau_g \qquad (32)$$

where τ_g is defined as:

$$\tau_g = J^T(F_g^d - K_I \int e_F dt) \qquad (33)$$

where $e_F = F_g - F_g^d$, F_g^d is the desired value of F_g and the gain matrix K_I is chosen as the diagonal and positive elements.

4. AVSC for the manipulator system

In the modal space, a modal control force F_m is used to suppress the vibration of the flexible link.

According to Eq. (18), the vibration equation becomes:

$$\ddot{Q} + C\dot{Q} + KQ = f_2 \qquad (34)$$

where $C = N_2^{-1}N_4$ is the generalized damping matrix, $K = N_2^{-1}N_5$ is the generalized stiffness matrix and $f_2 = -N_2^{-1}(N_1\ddot{X}_c + N_3\dot{X}_c + d_3)$ is the modal excitation force.

The modal control force $F_m = [F_{m1}\ F_{m2}]^T$ is introduced to the system, so Eq. (32) becomes:

$$\ddot{Q} + C\dot{Q} + KQ = f_2 + F_m \qquad (35)$$

Setting $x = \begin{bmatrix} Q & \dot{Q} \end{bmatrix}^T$, $u_2 = f_2 + F_m$. Eq. (35) becomes:

$$\dot{x} = Ax + Bu_2 \qquad (36)$$

where $A = \begin{bmatrix} 0_{2\times2} & I_{2\times2} \\ -K & -C \end{bmatrix}$, $B = \begin{bmatrix} 0_{2\times2} \\ I_{2\times2} \end{bmatrix}$.

A quadratic performance index J^* is defined as:

$$J^* = \frac{1}{2}\int_0^\infty (x(t)^T Wx(t) + u_2(t)^T Ru_2(t))dt \qquad (37)$$

where W is a positive-definite matrix and R is a positive matrix. The control objective is to design the optimal control law u_2 that minimizes the performance index J^*.

Then, the optimal control input $u_2(t)$ can be derived as:

$$u_2(t) = -R^{-1}B\overline{P}x(t) \qquad (38)$$

where matrix $\overline{P}$ is the unique positive symmetric solution to the Riccati equation and denotes:

$$-\overline{P}A - A^T\overline{P} + \overline{P}BR^{-1}B^T\overline{P} - W = 0 \qquad (39)$$

According to Eq. (35), the modal control force F_m is given by:

$$F_m = u_2 - \tilde{f}_2 \qquad (40)$$

where $\tilde{f}_2$ is the estimate of f_2. It is assumed that $\tilde{f}_2 = \tilde{N}_2^{-1}(\tilde{N}_1\ddot{X}_c + \tilde{N}_3\dot{X}_c)$, $\tilde{N}_1, \tilde{N}_2, \tilde{N}_3$ are the estimates of N_1, N_2, N_3. Eq. (40) becomes:

$$F_m = -R^{-1}B\overline{P}x(t) - \tilde{N}_2^{-1}(\tilde{N}_1\ddot{X}_c + \tilde{N}_3\dot{X}_c) \qquad (41)$$

The proposed method, which combines NTSMC with the modal force control, can guarantee the trajectory tracking and the vibration suppression.

According to Eq. (17), Eq. (18) and Eq. (35), the dynamic Eq. (19) can be written as:

$$A_k\ddot{X}_c + B_k\dot{X}_c + C_k\dot{Q} + D_kQ + E_k + H_2F_m = \tau - \tau_g \qquad (42)$$

Let's define the composite controller as τ:

$$\tau = A_k u_1 + \tau_g + H_2F_m \qquad (43)$$

The architecture of the composite control for the free-floating space rigid-flexible coupling manipulator system is shown in Fig. 4.

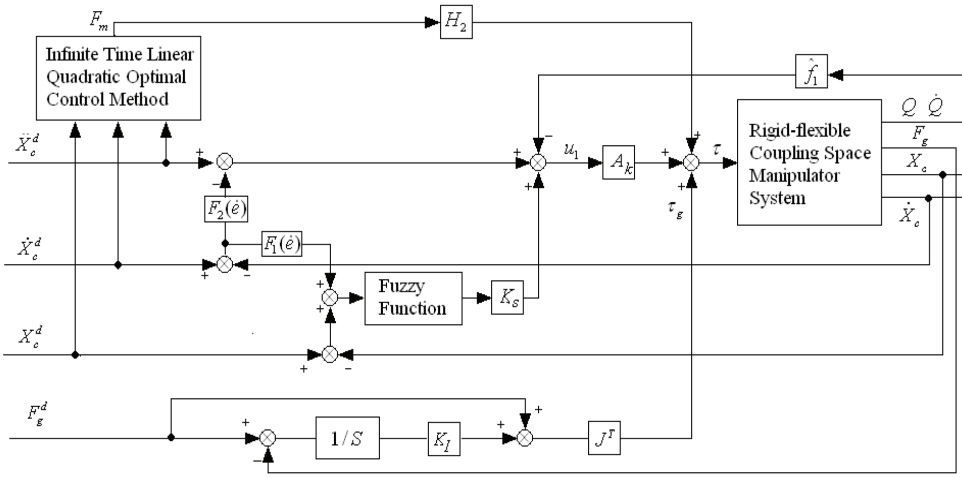

Figure 4. The architecture of the composite control for the free-floating space rigid-flexible coupling manipulator system

5. Simulation results

In this section, to illustrate the effectiveness of the proposed strategy, the numerical simulations have been performed via Matlab with SIMULINK. The parameters of the system are given in Table 1.

Link i	Mass(kg)	Length(m)	Inertia moment (kg·m²)	Module of elasticity (N·m²)
The base	2000	1.5(radius)	100	null
Link 1	2	2	1.2	null
Link 2	2	2	1.2	null
Link 3	2	2	0.2	200
The payload	20	0.5(radius)	15	null

Table 1. The parameters of the free-floating space rigid-flexible coupling manipulator system

The original pose of the base is set to $X_b = \left[1.5m \;\; 0m \;\; 0rad\right]^T$. The desired grasping force/moment is chosen as $F_g^d = \left[10N, 0N, 0N \cdot m\right]^T$. The model errors and disturbances $d1, d2, d3$ are assumed as $d1 = 0.1sgn(\dot{\theta})$, $d2 = \left[0.01\cos(5t) \;\; 0.01\cos(5t) \;\; 0.01\cos(5t)\right]^T$ and $d3 = 0.01sgn(\dot{Q})$.

Let $\hat{f}_1 = 0.8 f_1$, $p = 5$, $q = 3$, $K_s = 5$, $K_e = 2E_3$, $K_I = 5E_3$, $K_d = 6E_3$, $K_p = 10E_3$, $W = 6E_4$ and $R = 1.5E_2$, where $E_3 \in R^{3\times3}$, $E_4 \in R^{4\times4}$ and $E_2 \in R^{2\times2}$ are the unit matrices.

The desired trajectories for the payload are given by:

$$X_c^d = \begin{bmatrix} x_c^d(t) \\ y_c^d(t) \\ \theta_c^d(t) \end{bmatrix} = \begin{bmatrix} 2.5 + 0.1\cos(2t)(m) \\ 4 + 0.1\sin(2t)(m) \\ 0.1\cos(2t)(rad) \end{bmatrix}.$$

For the purpose of comparison, the simulations have been carried out by considering SMC and NTSMC for the control of the manipulator system in this work. The first is the classical SMC that shows the simulation results in Fig. 5-7. Fig. 8-10 show the simulation results under the controller NTSMC. Fig. 5 and 8 show the position curves of the base, with SMC and NTSMC, respectively. Fig. 6 and 9 denote the position tracking curves of the payload with SMC and NTSMC, respectively. Fig. 7 and 10 show the position tracking error curves of the payload with SMC and NTSMC, respectively. The simulation results show that the speed of the tracking error's convergence to zero for the NTSMC is faster.

Fig.11 shows the change curve of the modal force. Fig. 12 shows the change curve of the generalized coordinates of the flexible link. Fig. 13 shows the change curve of the grasping force between the end-effector and the payload. The above composite control scheme can track the expected trajectory accurately, even though there is uncertainty in the model parameters. The simulation results clearly illustrate that the tracking effect of the grasping force and the vibration suppression are very good.

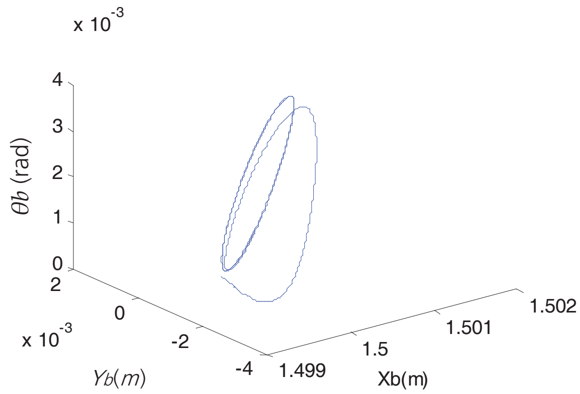

Figure 5. The position curves of the base with SMC

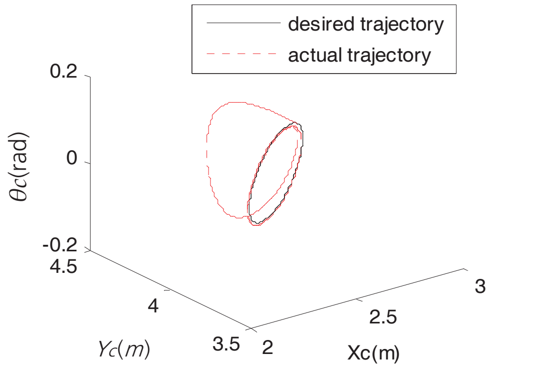

Figure 6. The position tracking curves of the payload with SMC

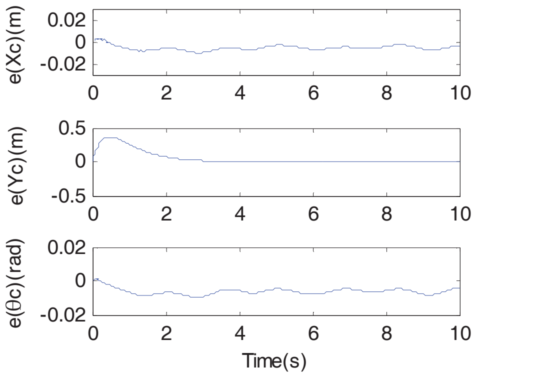

Figure 7. The position tracking error curves of the payload with SMC

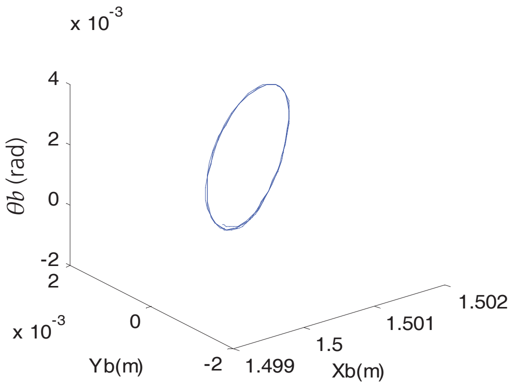

Figure 8. The position curves of the base with NTSMC

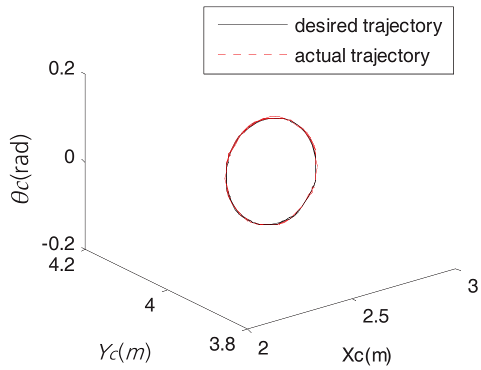

Figure 9. The position tracking curves of the payload with NTSMC

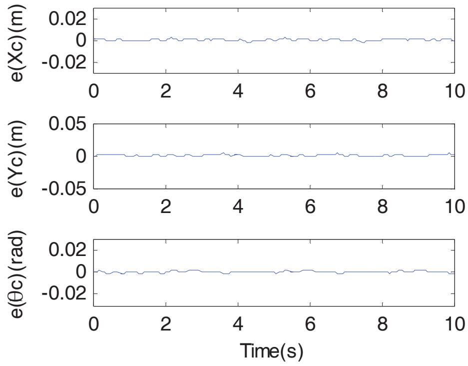

Figure 10. The position tracking error curves of the payload with NTSMC

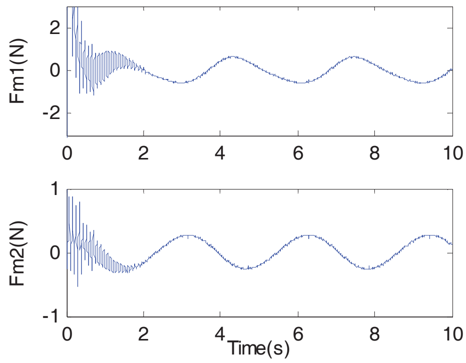

Figure 11. The change curves of the modal force

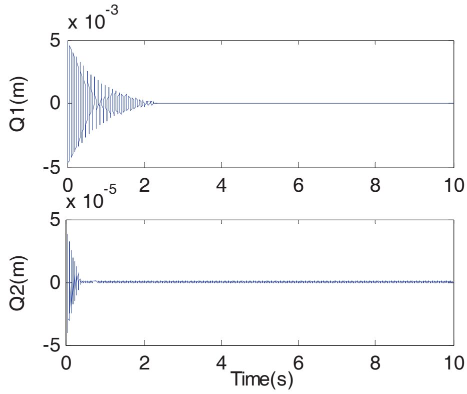

Figure 12. Generalized coordinates of the flexible link

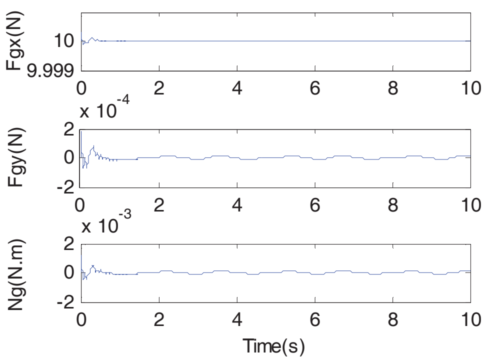

Figure 13. The change curves of the grasping force between end-effector and the payload.

6. Conclusions

The dynamic model is established by using an assumed mode method combined with Lagrange and momentum conservation theorem, while considering the modelling errors, external disturbance and vibration damping of the flexible link. Based on this model, SMC and NTSMC are designed to ensure the trajectory tracking control of the rigid payload, respectively. In the NTSMC controller, the fuzzy output is used, instead of the symbol item, to smooth the control signal, thereby the chattering of the sliding mode control has been inhibited. The NTSMC not only can guarantee the system states reach the terminal sliding surface faster, but also can eliminate the singular phenomenon for the system's control input. The modal force control is presented to suppress the flexible vibration, which is solved by using an infinite time linear quadratic optimal control method. The computer simulations have been performed to validate the effectiveness of the proposed controller, which demonstrate the proposed controller not only can assure the trajectory tracking, but also can suppress the flexible vibration and control the grasping force between the end-effector and the payload.

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8. References

[1] P. C. Hughes, Spacecraft pose dynamics, Wiley, Novel York, (1986).

[2] Y.R. Hu, G. Vukovich, Modeling and control of free-flying flexible joint coordinated robots, Advanced Robotics. ICAR '97. Proceedings 8th International Conference on (7-9, Jul, 1997), (1997) 1013-1020.

[3] P. W. Likins, Analytical dynamics and non- rigid spacecraft simulation, (JPL Technology Report, July), 1974.

[4] L.C.Wu, F.C.Sun, Z.Q. Sun, W.J. Su, Dynamic modeling, control and simulation of a flexible dual-arm space robot, TENCON '02. Proceedings 2002 IEEE Region 10 Conference on Computers, Communications, Control and Power Engineering, (2002) 1282-1285.

[5] Z. F. Yu, Y. B. Yu, D.Y. Shang,F. N. Yu, On orbit servicing flexible space robots dynamics and control during capturing targets, International Conference on Measuring Technology and Mechatronics Automation, Changsha 13(14), (2010) 817-820.

[6] M. Shibli, Unified modeling approach of kinematics, dynamics and control of a free-flying space robot interacting with a target satellite, Intelligent Control and Automation 2 (1), (2011) 8-23.

[7] N. Kumar, V. Panwar, N. Sukavanam, Neural network control of coordinated multiple manipulator systems, Proceedings of the International Conference on Computing: Theory and Applications (ICCTA'07), (2007) 250-256.

[8] H. A. Talebi, K. Khorasani, R. V. Pate, Neural network based control schemes for flexible–link manipulators: Simulations and experiments, Neural Networks.11(08), (1998) 1357–1377.

[9] Z. Vafa, S. Dubbowsky, The kinematics and dynamics of space manipulators: The virtual manipulator approach, International Journal of Robotics Research 9(4), (1990) 3-21.

[10] C. J. Damaren, Adaptive control of flexible manipulators carrying large uncertain payloads, Journal of Robotic Systems 13(4), (1996) 219-228.

[11] M. K. Kwak, H. Seok, G. J. Jin, Adaptive positive position feedback controller design for the vibration suppression of smart structures, Proc. of the 9th SPIE Conference on International Symposium on Smart Structures & Materials, Seattle (July), (2002) 35-40.

[12] I. Lizarraga, V. Etxebarria, Combined PD-H∞ approach to control of flexible manipulators using only directly measurable variables, Cybernetics and System 34(1), (2003) 19–31.

[13] T. Ravichandran, G. K. H. Pang, D. Wang, Robust H1 optimal control of a single flexible link, Control-Theory and Advanced Technology 9(4), (1993) 887-908.

[14] R. J. Theodore, A. Ghosal, Robust control of multilink flexible manipulators, Mechanism and Machine Theory 38(4), (2003) 367-377.

[15] F. L. Lewis, S. Jagannathan, A. Yesildirek, Neural network control of robot manipulators and nonlinear systems (Taylor and Francis, UK), 1999.

[16] A. Levant, Universal SISO sliding-mode controllers with finite-time convergence, IEEE Trans. Automat. Control 46, (2001) 1447–1451.

[17] V. Utkin, J. Guldner, J. Shi, Sliding modes in electromechanical Systems, London, UK, Taylor and Francis, (1999).

[18] L. Fridman, Sliding mode control for systems with fast end-effectors: Singularly perturbed, Variable Structure Systems: Toward the 21st Century 274, (2002) 391–415.

[19] A. Poznyak, Y. Shtessel, Mini–max sliding mode control with minimal time reaching phase, Proc. America, Control Conf., Denver, CO, (2003) 174–179.

[20] C-K Lin, Nonsingular terminal sliding mode control of robot manipulators using fuzzy wavelet networks, Fuzzy Systems, IEEE Transactions 14(6), (2006) 849-859.

[21] Y. Feng, X. H. Yu, Z. H. Man, Non-singular terminal sliding mode control of rigid manipulators, Automatica 38(12), (2002) 91–415.

[22] T.H.S. Li, Y.C. Huang, MIMO adaptive fuzzy terminal sliding-mode controller for robotic manipulators, Information Sciences.180, (2010) 4641–4660.
[23] M. R. Soltanpour, M. M. Fateh, Sliding mode robust control of robot manipulator in the task space by support of feedback linearization and backstepping control, World Applied Sciences Journal 6(1), (2009) 70-76.
[24] J. H. Lee, M. J. Youn, A novel integral variable structure regulation controller for robot manipulators with accurately predetermined output, ISIE'99, Bled, Slovenia, (1999) 336-341.
[25] K. Nagai, T. Yoshikawa, Adaptive grasping by multifingered hands, Journal of the Robotics Society of Japan 11(1), (1993) 111-120.
[26] Y. Kazuya, Space robot dynamics and control: to orbit, form orbit and future robotics research, Proceedings of the 9th International Symposium, Snowbird, UT; UK,(1999) 449-456.
[27] H. Ying, A nonlinear fuzzy controller with linear rules is the sum of a global two-dimension multilevel relay and a local nonlinear proportional-integral controller, Automatica 29(2), (1993) 499-505.